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SHORT-TERM FORECASTING OF SLOVENIAN GDP USING MONTHLY INFORMATION

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SHORT-TERM FORECASTING OF SLOVENIAN GDP USING MONTHLY INFORMATION*

Jan Radovan†

ABSTRACT

This paper presents the pseudo out-of-sample now/forecasting exercise for Slovenian quarterly GDP at one and two-quarter ahead horizons by implementing five standard and commonly used forecasting models. Based on the progression of the exercise, we arrive at several interesting conclusions. First, models that use additional monthly data outperform the benchmark AR model that is based solely on the information content in real GDP growth, and the forecasts obtained from the models that use additional monthly information tend to be increasingly more accurate when observing a particular quarter. Second, considering all horizons, medium-scale version of the DFM (MDFM) produces the most accurate one and two-quarter ahead forecasts, indicating that it is feasible to efficiently utilize larger information sets within the factor analysis framework. Third, the availability of monthly indicators for the current period is found to be the most promising at obtaining more accurate short-term forecasts of real GDP growth. Lastly, we can conclude that in the case of Slovenia smaller data sets (MDFM) are found to perform better and yield superior short-term forecasting performance in comparison to larger alternatives (LDFM).

POVZETEK

V članku predstavljamo psevdo izven-vzorčne kratkoročne napovedi slovenskega četrletnega BDP za obdobji enega in dveh četrletij vnaprej, pri čemer primerjamo napovedne moči petih najbolj standardnih in pogosto uporabljenih modelov. Na podlagi dobljenih rezultatov, oblikujemo nekaj zanimivih zaključkov. Prvič, modeli, ki pri napovedovanju uporabljajo dodatne mesečne indikatorje prekašajo primerjalni AR model, ki temelji izključno na informacijah, prisotnih v rasti realnega BDP. Ravno tako se natančnost prej omenjenih modelov znotraj opazovanega četrletja povečuje predvsem zaradi možnosti izkoristka dodatnih informacij. Drugič, srednje velika različica DFM (MDFM) izkazuje najnatančnejše napovedi tako za obdobji enega kot dveh četrletij vnaprej, kar nakazuje na učinkovitost uporabe večjih informacijskih množic znotraj okvirov faktorske analize. Tretjič, razpoložljivost mesečnih indikatorjev za tekoče obdobje se izkaže kot najobetavnejša pri pridobivanju bolj natančnih kratkoročnih napovedi rasti realnega BDP. Zadnja od ugotovite prinaša sklep, da je v primeru Slovenije uporaba manjših podatkovnih sklopov (MDFM) učinkovitejša in prinaša superiorno kratkoročno napovedno moč v primerjavi z večjimi alternativami (LDFM).

JEL Classification: C22, C32, C38, C52, C53, E27

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1 Introduction

Forecasting macroeconomic variables (especially gross domestic product or simply GDP) is a very relevant and challenging topic of the current macroeconomic research. The correct assessment of the general economic conditions is especially important in the area of policy making, since spillover effects of inadequate and untimely policy actions could deliver unfavourable effects on the economy. One of the major goals of empirical studies is therefore to develop a method that is able to describe short-term fluctuations in economic activity (GDP is treated as a reference series) as accurately as possible in order to earlier detect negative developments in the macroeconomic environment. Beside the aforementioned argument, most recent predictions are usually the most relevant, since accuracy of forecasts is typically better in the short-run than in the long-run. This is one of the main reasons why short-term forecasting plays a significant role in decision making process.

Regarding the short-term forecasting methods, recent econometric research is mainly focused on solving issues related to specific structures of data and resolving problems associated with existence of different timing of data releases of numerous series. When building a quarterly forecasting framework, it is often the case that we arrive at the problem of constructing a model with variables sampled at different frequencies, where GDP is usually reported at quarterly frequency, while most of other economic and financial data are accessible at monthly or even daily frequencies. In addition, significant complications are also caused by lags in data availability of macroeconomic variables, since some series are released with significant delays. With regard to the mixed frequencies issues, there exist two commonly used solutions, one following the approach proposed by Mariano and Murasawa (2003) (approach also adopted by Giannone et al., 2008), and the other called quadratic (or linear) interpolation of quarterly GDP to monthly level (approach adopted by Liu et al., 2011 and Matheson 2011). In our research we decided to choose the latter since it is able to solve two problems at the time; first, the issue of frequency mismatch disappears and second, the amount of information loss is significantly reduced although some degree of loss still exists due to transformation of daily data to monthly averages. With all time series sampled at the same frequency, it is possible to build a macroeconomic model for short-term real GDP growth forecasting.

In this paper, the dynamic factor model (DFM) approach is proposed (similar to Doz et al., 2011), since the majority of central banks have lately shown an increasing interest in using DFM to produce short-term real GDP growth forecasts¹. We utilize monthly releases of time series in order to be able to exploit contribution of additional information content of more frequently released variables and by that examine their potential effect on the precision of

¹More detailed description of model is available in Section 3.

the short-term forecasts. In such manner, the advantages of DFM approach become crucial since they allow to include a large number of monthly time series indicators and on the other side also efficiently handle unbalanced data sets caused by non-synchronous publication lags. Usefulness of larger information sets at forecasting is confirmed by several papers (for instance Boivin and Ng, 2005; Forni et al., 2005; Giannone et al., 2004; Marcellino et al., 2003) and they all recommend and employ factor models (FM), adapted to handle large data sets. The primary goal of the current paper is therefore to test a series of relevant models for short-term forecasting of domestic real GDP growth and compare their forecasting performance. In that sense we could answer the question whether a larger information set really helps obtaining more accurate short-term (one and two-quarter ahead) forecasts of Slovenia output growth. To the best of our knowledge, current research is one the first applications of such up-to-date methods for Slovenian data which also compares DFM's forecasting performance to the most common alternative models, therefore making the paper fairly relevant for the Bank of Slovenia.

The rest of the paper is organized in a following manner. Section 2 presents a short literature review of the most important findings of previous research. Section 3 discusses DFM modelling procedure that stands behind the DFM theory and provides main characteristics of alternative (competing) models used in the paper. Section 4 presents the data set for Slovenian economy, the applied statistical techniques that are necessary to modify the data and demonstrates the results of pseudo out-of-sample forecasting exercise, where special attention is given to comparing the forecasting performance. Finally, Section 5 concludes with the main findings.

2 Literature review

First, we review the latest and most important literature on short-term forecasting. In each of the corresponding forecasting research, special attention is given to the econometric methodologies used and their main findings. Most of the review focuses on the DFM theory, since it has recently received considerable attention among the central bankers, mainly due to its ability to simultaneously and consistently handling large and "ragged edge" (induced by various publication lags) data sets. However, it should be pointed out that in our process of pseudo out-of-sample forecasting exercise an assumption is made that at the moment of the projection all the chosen economic indicators are readily available which leaves us with the realigned and balanced set of monthly indicators (similar to approach in Altissimo et al., 2010).

Even before the existence of the present-day problems, researchers have been challenged with the distinctive characteristics of macroeconomic data sets, especially issues related to the short samples. Early works of Geweke

(1977,) and Sargent and Sims (1977) proposed a solution in form of frequency-domain DFM, which has the ability to model data sets that has more number of series than there are number of time series observations. Especially Sargent and Sims (1977), whose main focus was to look for evidence of a dynamic factor structure² and to estimate the importance of that factor, discovered that small number of acquired latent factors is often enough to explain large portion of the variability, present in the given macroeconomic data set. Those early methods, however, could not estimate factors directly and thus could not be used for forecasting, meaning that all subsequent research was focused on time-domain methods, since they enable direct estimation of the latent factors using the Kalman filter algorithm.

Research developments regarding the time-domain estimation of DFMs can be according to Stock and Watson (2011, 2016) divided into three stages (generations). The first generation of research (Engle and Watson, 1981, 1983; Stock and Watson, 1989; Sargent, 1989) consisted of low-dimensional parametric models (models with small number of time series) that used Gaussian maximum likelihood estimation (MLE) in order to estimate parameters and the Kalman smoother (filter) procedures to obtain efficient estimates of the factors. An advantage of such approach is that it can handle data irregularities, but on the other side requires non-linear optimization, which in most cases restricts the number of parameters and hence the number of series that could be handled. In order to deal with the issues of the first generation, the second generation of estimators entailed non-parametric estimation with large number of time series using cross-sectional averaging (shrinking) methods, primarily principal components (PC) and other related methods. The key finding of the second generation is that PC (Connor and Korajczyk, 1986; Stock and Watson, 2002)³ and other related estimators (Forni et al., 2005) of the factors are consistent. Moreover, if the number of observations is sufficiently large, Bai and Ng (2006) showed that factors can be estimated precisely enough to be treated as data in regressions. The last generation uses contributions of previous research and applies second generation's consistent non-parametric estimates of the factors to the first generation's state-space model parameter estimation procedure and thereby solves the dimensionality problem associated with first generation models. Such technique has nowadays been also extensively used in the macroeconomic field of forecasting.

When talking about the modern i.e. third generation DFM framework, the paper by Giannone et al. (2008), which uses large US time series data set

²The main focus of the modelling procedure was put on dynamic statistical procedures behind the core of the DFM theory that form a basis upon which latent dynamic factors can be obtained.

³Connor and Korajczyk (1986) showed the consistency of PC estimator in the exact factor model, while Stock and Watson (2002) proved uniform consistency of the factors by using Chamberlain and Rothschild's (1983) approximate factor model.

with varying frequencies and release dates, is a major reference. Their DFM is presented in state-space form, and is estimated in two steps using the Kalman smoother and filter algorithms (approach first developed by Doz et al., 2011) to deal with mixed frequencies and unbalanced data set. An important advantage of the framework is that it can accommodate a potentially large number of variables by summarizing the information in a few common factors. The main results of the analysis show that the accuracy of the quarterly real GDP growth forecasts evolves and improves with the availability of new monthly data (positive marginal impact of news in each data release), on top of which the sources of the changes in the forecast due to timeliness of information vs. due to economic content can be traced out.

Similar findings are also obtained by researches using data sets of other countries or groups of countries⁴. Angelini et al. (2011), who use a large EA time series data set, closely follow the approach developed by Giannone et al. (2008) and find out that DFM produces superior forecasting accuracy in comparison to the traditional bridge equation model (BE). They also show that the impact of softer indicators is more useful at the beginning of the quarter while that of the hard indicators at the end of the quarter. Bańbura and Rünstler (2011), and Bańbura and Modugno (2014) use alternative DFM estimation technique called quasi maximum likelihood developed by Doz et al. (2012) but obtain similar conclusions as Giannone et al. (2008). In addition, Bańbura and Modugno (2014) also demonstrate how to extract a model based news from statistical data release, and how to derive the relationship between the news and resulting forecast revision. Liu et al. (2011) put the Giannone et al. (2008) framework in the context of a panel of Latin America economies as one of the first research that focuses on EMEs, and compare the forecasting performance of DFM and five alternative models (AR, pooled BE, pooled bivariate VAR, and Bayesian VAR or BVAR). They find out that; first, models that use monthly data generally outperform the quarterly AR model and that the flow of monthly data releases is important; second, DFM produces more accurate forecasts relative to other models across most countries considered; third, external indicators are useful in improving the precision of forecasts for most Latin American countries.

On the other side, few authors also employ small-scale DFMs in order to show that more indicators do not necessarily always lead to higher forecasting accuracy⁵. The findings of the papers are also highly relevant for our case, since a lot of Slovenian time series are too short to be included in the research, leaving us with a small set of relevant indicators. For EA, Camacho and Perez-Quiors (2010) construct a small-scale (13 variables) DFM to forecast EA real GDP growth and find out that their predictions usually perform better than professional forecasts of different institutions. In addition, they show that flash

⁴For an extensive list of references see Bańbura et al. (2011) and Bańbura et al. (2013).

⁵For an extensive list of references see Camacho et al. (2013).

announcements and business surveys lead to a reduction in forecasting uncertainty. Regarding the research using US data, Camacho and Martinez-Martin (2014) adapt a small-size DFM based on the research of Aruoba and Diebold (2010), where authors also include financial or so called leading indicators and survey data. Results of their exercise show similar findings as are obtained by Aruoba and Diebold (2010) but far better than the conclusions of the basic models like AR and random walk (RW) which is true especially in recessions.

The current research could also be put in the context of related studies from Central, Eastern and South-Eastern Europe countries (CESEE). Their findings may be beneficial, as data limitations and data selection procedures in the neighbouring countries are very similar to those observed in Slovenia. For Czech Republic, Arnořtová et al. (2011) find that a medium-size static principal components (PC) model and a DFM as in Doz et al. (2011) are the best performers among all the competing models. However, it should be pointed out that once the authors use the full set of monthly indicators, the factor model (FM) estimates turn out worse meaning that smaller FM tend to display better forecasting performance. Franta et al. (2014) step even further and compare the DFM methodology of Bańbura and Modugno (2014) to most common alternative short-term forecasting models (mixed data sampling regression models i.e. MIDAS and mixed frequency (B)VAR i.e. MF-(B)VAR). They find that short-term performance of DFM is comparable to the forecasts published by the CNB, while at longer horizons, MF-VARs are more suitable, especially MF-BVAR. In the case of Slovakia, Huček et al. (2015) reveal that small-scale FMs outperform an ARMA model and are also able to compete with BE model, while for Russia, Porshakov et al. (2015) show that a large-scale DFM procedure of Doz et al. (2011) is able to outperform simple competing models, like RW and BE. However, in contrast to aforementioned studies, Porshakov et al. (2015) find that larger DFM is more accurate than smaller-scale versions at one and two-quarter ahead predictions. Finally, results of the group of CESEE countries (Feldkircher et al., 2015) reveal that small-scale forecasting models have a clear advantage over purely time series based estimates (AR model), while in comparison to different types of BEs, DFM is not a favourite for all countries.

A very common alternative approach to DFM, developed recently are MIDAS and MF-(B)VARs⁶. In MIDAS approach, initially proposed by Ghysels et al. (2004, 2006), dependent and independent variables are sampled at different frequencies and enter directly into the regression without any pre-specified form of aggregation⁷. This method has recently been employed to forecast macroeconomic time series and results show that introduction of monthly or even daily time series improve quarterly forecasts (Tay 2006; Clements and Galvão,

⁶For more detailed treatment of alternative methods see Foroni and Marcellino (2013).

⁷Regressors are usually defined at a higher frequency, while, in order to ensure parsimonious representation of the model, distributed lag polynomials are used.

2008; Armesto et al., 2010; Andreou et al., 2013). In comparison to DFM, MIDAS does not require to explicitly specify a linear dynamic model for all the included series and can be therefore treated as a substitute for the Kalman filter. In case of MF-(B)VARs, the state-space representation of the model is used where the low frequency variable is treated as a high frequency one with missing observations, while the Kalman filter and smoother are applied to estimate the missing observations and to generate forecasts (Mariano and Murasawa, 2010). Recent contributions can be divided according to whether the system of equations is estimated via ML (Kuzin et al., 2011) or Bayesian approach (Bańbura et al., 2010; Schorfheide and Song, 2015).

The current paper is one of the first to close the existing gap in Slovenian short-term forecasting literature and could therefore present value-added for the Bank of Slovenia. It focuses on the already mentioned third DFM generation methods and applies them to Slovenian short-term forecasting study. In that sense results of the exercise reflect the most up to date findings, issues and tendencies related to the field of now/short-term forecasting.

3 Modelling methodology

In the present paper, the DFM approach is used, since the majority of central banks have lately shown an interest in using it to produce short-term real GDP growth forecasts. DFM can be treated as an extension or generalization of the static factor analysis, which is found to be especially helpful for forecasting macroeconomic environment. The first part of this Section describes the theory behind the DFM. It is based on dynamic statistical procedures that form a basis upon which latent dynamic factors can be obtained. We will see that it is usually enough to exploit just a small number of factors to be able to explain a great significant portion of the variability present in a given macroeconomic data set. In the following step, we also want to evaluate the forecasting performance of DFM. To be able to compare the accuracy of forecasts among different methodological ideas some commonly used alternative models should be included in the analysis. The second part of the Section hence presents their main modelling characteristics.

3.1 DFM theory

The main aim of the study is to develop a short-term forecasting model of GDP in a state-space form, using a mixed frequency data set. The use of state-space modelling has recently become widely accepted to present dynamic structural relationships between unobserved components i.e. extracted factors and a (potentially large) set of observable macroeconomic series. The whole idea of state-space modelling is simply to relate two equations in a single system, where the first equation is called the signal equation while second, which is not

directly observable, is called the state equation. Research mentioned in Section 2 already demonstrated how to combine the DFM theory with state-space modelling approach into one consistent concept to obtain proper forecasts of domestic real GDP growth. What becomes crucial is the background of the DFM theory for which the state-space representation is highly convenient.

In its essence, the DFM modelling procedure suggests that a large macroeconomic database can be split into two mutually orthogonal and unobserved components; common and idiosyncratic. The common component is composed out of few unobserved factors that drive the dynamics of a large sample of time series variables and therefore captures the bulk of the covariation between series, where on the other side the idiosyncratic component arises from measurement error and from specific characteristics of individual series (Stock and Watson, 2011). The idea of dynamic factor analysis is hence based on the concept of patterns of change also known as factors or unobservable phenomena, which are common to all variables and are determined from the covariance matrix of a large set of indicators that display the variability of the set. Because of their special characteristic and statistical construction, factors do not have clear economic interpretation and are often believed to be the driving forces of the economy (Stock and Watson, 1998). In DFM, latent factors usually follow a VAR(p) process. The mathematical representation of the multivariate state-space form of the model can be illustrated with the following two equations defined at the monthly frequency:

$$X_t = \Lambda F_t + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \Psi) \quad (1)$$

$$F_t = \sum_{s=1}^p A_s F_{t-s} + B u_t, \text{ where } u_t \sim N(0, \Sigma) \quad (2)$$

where both ε_t and u_t are considered as independent Gaussian errors. The first (signal) equation relates the $k \times 1$ vector of observed monthly indicators X_t that includes also quadratically interpolated real GDP growth to the $r \times 1$ vector of common factors F_t (state variables) via the factor loadings Λ i.e. coefficients of the linear combinations of the factors, accounting also for the idiosyncratic component (measurement error) ε_t . We additionally allow for correlation between idiosyncratic (specific) components i.e.

$$\text{cov}(\varepsilon_{i,t}, \varepsilon_{j,t}) \neq 0 \text{ for all } i \neq j, \quad (3)$$

which means that we define an approximate factor model (Chamberlain and Rothschild, 1983). The second equation of the specified system displays the time evolution of the state variable, meaning that the common factors follow a VAR(p) process which is driven by a $q \times 1$ vector of pervasive shocks u_t .

By using a state-space form we can recover the estimates of the unobservable state variables from the observable series using the Kalman smoother and filter algorithms⁸. Lastly, it should be mentioned that the factors are obtained using the most common method called the principal components analysis (PCA). It was first introduced by Stock and Watson (1998), and it gained popularity due to its computational simplicity and well established asymptotic properties of its estimator under some general assumptions.

Regarding the details about the estimation procedure, DFM is further estimated using the two-step procedure described in Doz et al. (2011):

1. Based on the latest available complete data set, we estimate the common factors using the PCA method. Given the common factors, we estimate the factor loadings $\hat{\Lambda}$ and the covariance matrix $\hat{\Psi}$ associated with ε_t using standard OLS procedure. In addition, we also have to estimate the VAR coefficients $\hat{A}_1, \dots, \hat{A}_p$ and $\hat{\Sigma}$ using the OLS procedure, where the number of lags p is determined using the Schwartz Information Criterion (SIC);
2. Given the estimated parameters ($\hat{\Lambda}, \hat{\Psi}, \hat{A}_1, \dots, \hat{A}_p$ and $\hat{\Sigma}$) from step 1, we apply the Kalman smoother to the entire data set, including missing observation, and re-estimate the factors. This smoothing procedure is extremely convenient, since the implicit signal extraction process of the filter places no weight on the missing variable in X_t , when computing the factors at time t .

Doz et al. (2011) have shown that the two-step procedure outlined above gives consistent estimates of the factors⁹. In the final step, we apply the Kalman filter forward recursion using the estimated factors from step 2 to obtain h -step ahead forecast for real GDP growth.

3.2 Specification of some commonly used competing models

To objectively judge the forecasting performance of DFM, a set of alternative models is also considered. They are carefully chosen on the basis of the analyses mentioned in the Literature review and range from a simple AR process to more sophisticated BVAR and PC models. Their selection is in some way also suitable for dealing with medium or even larger data sets. The models considered here are only a small subset of vast amount of competing methods,

⁸More detailed treatment of Kalman filter algorithm, its filtering and smoothing procedures is presented in Hamilton (1994).

⁹It is also worth noting that similar reasoning was applied in a separate paper by Doz et al. (2012), where authors showed that by iterating steps 1 and 2, a quasi maximum likelihood estimator for the factors is obtained).

but they also represent a common set of tools used in many policy making institutions, such as central banks.

3.2.1 AR model

As a benchmark, a simple univariate AR model of order p is used for real GDP growth (y_t):

$$y_t = c + \sum_{i=1}^p \beta_i y_{t-i} + \varepsilon_t, \quad (4)$$

where c is a constant, ε_t is a white noise term such that $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, and β_i are the parameters of the model. The lag length p is determined using SIC. However, it should be mentioned that due to short sample limitations, the AR(p) model is converted to monthly frequency (quadratic interpolation) in order to assure that estimation sample in pseudo out-of-sample forecasting exercise consists of more observations than in the quarterly case¹⁰. Since the baseline AR(p) model does not exploit any other monthly information flow, we treat it as a benchmark on the basis of which other models' performance will be evaluated.

3.2.2 Bivariate OLS VAR model

VAR models seem like a natural extension of simple univariate time series models, proven to be especially useful for forecasting. In our case we decide to work with a Bivariate OLS VAR model in order to be able to include a large set of variables and at the same time not to encounter issues of over-parameterization. Like BE, another widely used method for forecasting quarterly GDP by using monthly indicators, the Bivariate OLS VAR model also exploits the information content of monthly indicators but differs from the former method in an important manner. BE models usually operate in a way that they use autoregressive forecasts obtained from each individual monthly indicator in order to forecast quarterly GDP, while Bivariate OLS VAR models directly uses information in real GDP growth. This usually delivers some efficiency gains by better capturing the interrelated dynamics and hence produces more accurate forecasts of real GDP growth. Additionally, some of the dynamics between each of the monthly indicators and GDP are obtained, letting y_t^I denote interpolated quarterly GDP growth at the monthly frequency. The Bivariate OLS VAR(p) model on GDP growth (y_t^I) and each of the monthly indicators, $\{x_{1,t}, x_{2,t}, \dots, x_{k,t}\}$ is thus estimated using the following equation:

¹⁰We had considerable problems with the implementation of the quarterly version of the model, since its estimation and forecasting performance simply did not provide comparable results to the performance of other models. For that reason, we decide to convert the model to monthly frequency.

$$Y_{i,t} = c_i + \sum_{s=1}^{p_i} \beta_s Y_{i,t-s} + \varepsilon_{i,t}, \quad (5)$$

where c_i is a 2×1 vector of constant terms, $Y_t = [y_t^I x_{i,t}]'$ presents the 2×1 vector of real GDP growth and the corresponding variable, β_s are 2×2 coefficient matrices and ε_t is an 2×1 unobservable zero mean white noise vector process. The lag length of each model is as in the previous case determined using the SIC. The final forecast for real GDP growth is formed using the average of the forecasts from all pairwise models.

3.2.3 BVAR model

One disadvantage of VAR models is the so called "curse of dimensionality" problem that arises due to inclusion of too many variables and usually results in a rapid increase in the number of estimated reduced-form parameters. At the same time, the problem of over-parametarization can appear when we operate with short samples, since in most cases the OLS estimates of parameters turn out imprecise. In order to deal with the aforementioned problems we employ a BVAR model that is generally able to produce more precise estimates by incorporating prior information (distribution) of the parameters into the estimation process¹¹. By doing so we obtain additional information that reflects the researcher's pre-estimation knowledge and beliefs. In order to be able to mutually compare the results from Bivariate OLS VAR and BVAR models, we decided to contrast their performance in specifications where both models include the same set of variables. Using the same model notation as above Y_t now includes the same set of monthly indicators, as well as the quadratically interpolated real GDP growth:

$$Y_t = c + \sum_{s=1}^p \beta_s Y_{t-s} + \varepsilon_t. \quad (6)$$

In comparison to the Bivariate OLS VAR model, c is now a $k \times 1$ vector of constant terms, $Y_t = [y_t^I x_{1,t}, x_{2,t}, \dots, x_{k,t}]'$ presents the $k \times 1$ vector of the above mentioned set of time series variables, β_s are $k \times k$ coefficient matrices and ε_t is an $k \times 1$ unobservable zero mean white noise vector process. The lag length of the model is again determined using the SIC.

When setting the prior distribution, we choose among several methods discussed in Ciccarelli and Rebucci (2003). For the forecasting purposes¹² we

¹¹model parameters in $\theta = (\beta, \Sigma)$ are characterized by the distribution with some prior mean and prior variance.

¹²We run the BVAR model with several different prior specifications, each time using the same values for hyper-parameters, and find out that Minnesota prior produces the most accurate forecasts.

decided to follow the methodology based on early research of Doan et al. (1984) and Litterman (1986), where the authors developed a well known Minnesota prior¹³. This type of prior is based on the assumption that the variance of the error terms Σ_ε is fixed and diagonal, with its diagonal elements obtained from the estimation of a set of univariate AR models. The value of μ_1 which is an AR coefficient is set to be equal to 1. Given the characterized Σ_ε , the prior for the VAR coefficient θ is further assumed to be normal with its mean equal to θ_0 and variance Ω_0 specified in terms of three hyper-parameters that control overall tightness (λ_1), relative cross-variable weight (λ_2) and lag decay (λ_3). Once we increase the lag length, λ_3 governs the shrinking speed of off-diagonal elements of variance matrix towards 0. In our medium-scale BVAR model, we follow standard literature (for instance Canova, 2007) and the recent BVAR research for house prices forecasting in Slovenia (Lenarčič et al., 2016) to determine the values of the hyper-parameters. As is reflected in the following table, all three hyper-parameters are within the range of commonly used values:

Table 1: Values of hyper-parameters in Minnesota prior

| λ_1 | λ_2 | λ_3 |
|-------------|-------------|-------------|
| 0.25 | 0.45 | 1.00 |

Source: Canova, 2007; Lenarčič et al., 2016.

The main reason for choosing the Minnesota prior is to minimize the possibility of over-fitting the BVAR model, while at the same time the applied technique is very appealing due to its simplicity and success in different forecasting applications (Koop and Korobilis, 2010).

3.2.4 PC model

The last considered alternative is the so called PC model. Its modelling foundations were first developed by Stock and Watson (2002), and are in some sense fairly close to the DFM approach. The mathematical exposition of the PC model can be thus illustrated with the following two equations:

$$X_t = \Lambda F_t + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \Psi) \quad (7)$$

$$y_t = \beta'_F F_t + \beta'_w w_t + u_t, \text{ where } u_t \sim N(0, \Sigma) \quad (8)$$

In the Equation 7, X_t is the $k \times 1$ vector of monthly indicators that also include quadratically interpolated real GDP, F_t is $r \times 1$ vector of common (static)

¹³For more detailed treatment of underlying distributional assumptions and detailed derivations see Ciccarelli and Rebucci (2003).

factors, and ε_t is an $n \times 1$ vector of idiosyncratic disturbances. Regarding the Equation 8, y_t represents the particular series that we want to forecast (GDP growth), w_t is a $m \times 1$ vector of observed variables¹⁴, that together with F_t are useful for forecasting y_t , and u_t represents the OLS regression error term. β_F and β_w are the corresponding OLS coefficient matrices. As in the DFM case, we allow for serially and cross-sectionally correlated error terms ε_t , meaning that we define an approximate static factor model first introduced by Chamberline and Rothschild (1983).

The estimation procedure of the model, is implemented in two-steps. In the first step of the process, the PC model utilizes the same large data set of predictors as DFM, in order to extract a few unobservable common factors that drive the dynamics of the data set. In the next step PC model deviates from DFM in an important manner, since the estimated factors from PCA are, together with w_t , used in the Equation 8 to estimate OLS linear regression coefficients. Forecasts are finally constructed on the basis of both, the estimated factors (\hat{F}_t) and regression coefficients ($\hat{\beta}_F$ and $\hat{\beta}_w$).

3.3 Benefits and drawbacks of DFM approach

One of the biggest advantages of the DFM is undoubtedly its ability to handle large sets of variables and to exploit information content present in database, since including more information in a model may seem helpful in generating more accurate predictions. A traditional forecasting techniques, utilized in the form of univariate (AR) and multivariate (VAR) time series models, usually suffer due to several limitations. To be more specific, univariate models normally use only a limited subset of the whole information set, while multivariate models frequently experience the "curse of dimensionality", since estimating more parameters generally results in imprecise (inefficient) estimation due to the small number of degrees of freedom. In order to exploit as much information as possible without the problem of parameter inflation, some statistical techniques are required. DFM can thus be treated as a flexible extension of widely accepted time series models, since the PCA statistical method is able to handle large number of predictors without entering them directly into the model. At the same time, it avoids the question which variables should be abandoned from the data set of regressors in order to gain tractability. Due to the unique characteristics and atheoretical construction of factors, it is also possible to avoid any prior assumptions about the direction and shape of mutual economic relations.

A second advantage is, that state-space form presents a single system, whose primary benefit is to integrate unobserved components (state variables) with observable series. At the same time the above mentioned technique uses

¹⁴Most often own lags of y_t are used.

a recursive algorithm called Kalman filter in order to recursively update the state variables and to deliver h -step ahead forecast for real GDP growth.

The biggest drawback of the method is the lack of a clear economic interpretation of the factors due to their atheoretical construction. Factors are simply treated as patterns of change or unobservable driving forces which are common to all variables and therefore reflect the variability of the set of predictors. The long term problem related to such conceptual generalization is the inability of DFM to closely examine the impact of a specific regressor on the dependent variable. However, most recent developments in DFM theory (Bańbura and Modugno, 2014) suggest a method that is able to compute the contribution of news in each monthly indicator to the real GDP growth series, which makes the DFM approach more suitable for examining the structure of particular phenomena observed in the economy.

Lastly, we can say that its abstract nature can be considered as less transparent for researchers not familiar with quantitative methods of analysis. It is usually the case that state-space models are quite involved, as one must explicitly specify a linear dynamic model for every included series, which means that a large set of parameters is required, namely for the measurement equation, the state dynamics, and their error processes.

4 Empirical analysis

This Section of the paper presents the empirical analysis and its main findings. It is divided into two Subsections. The first part presents the data set for Slovenian economy, statistical techniques and data transformations applied. Furthermore, the selection procedure regarding the choice of relevant variables is described. The second part first presents the pseudo out-of-sample forecasting framework and the most common statistical accuracy measures used to compare forecasting performance of applied models. In the next stage the results of the pseudo out-of-sample forecasting exercise are presented and discussed. Special attention is given to the comparison of DFM's forecasting performance to some standard and commonly used alternative models.

4.1 Data

The data set used in the study consists of 74 time series describing the development of the most important aggregates of Slovenian economy and a few taking into account situation in external environment¹⁵. The complete database is only available from 2003 M7 to 2017 M1 (real GDP growth can be estimated up to 2016 Q3), as a result of inaccessibility of the longer time series for

¹⁵The full set of variables and their correlation with real GDP growth is available in Appendix A.

many of the indicators¹⁶. Essentially, the entire set of indicators is denominated in monthly or even higher frequency, except the real GDP growth for which the monthly values are obtained via the quadratic interpolation. All macroeconomic series are obtained from Statistical office of the Republic of Slovenia (SORS), Eurostat, and the Bank of Slovenia, and they cover five broader macroeconomic groups: real variables, money and financial market conditions, price movements, labour market and employment aggregates, and survey indicators¹⁷.

All the indicators under consideration are, before the estimation procedure, pre-adjusted in the following way. In the first step, TRAMO/SEATS seasonal adjustment method is used if the particular series exhibit seasonality¹⁸. The specifics of the included series are taken into account by using extensive set of TRAMO/SEATS options¹⁹. In the next step, each individual time series is transformed in order to obtain stationary processes by using one of the following four methods which were also applied by Giannone et al. (2008):

- No transformation $\Rightarrow y_{i,t} = Y_{i,t}$ (applied only to growth rate of real GDP);
- Three-month difference $\Rightarrow y_{i,t} = (1 - L^3)Y_{i,t}$ (applied mostly to survey indicators, and also to few financial and employment aggregates);
- Three-month growth rate $\Rightarrow y_{i,t} = (1 - L^3)\log Y_{i,t} \times 100$ (applied to real variables, most money market and financial indicators, and employment aggregates);
- Three-month difference of yearly growth rate $\Rightarrow y_{i,t} = (1 - L^3)(1 - L^{12})\log Y_{i,t} \times 100$ (applied to prices in order to take into account three-month difference as well as seasonal difference at $s = 12$).

Finally, in order to make sure that only the most relevant monthly indicators enter the DFM, we obtain the matrix of correlation coefficients for every series and the real GDP growth, and apply a simple shrinkage rule to the full

¹⁶Wage indicators are completely excluded, as they are only available from 2005 M1 onwards.

¹⁷All considered groups include also exogenous indicators.

¹⁸All series except survey indicators, ECB main refinancing rate, exchange rates, balance of payments data and financial market indices are seasonally adjusted.

¹⁹TRAMO/SEATS options incorporate: test for log-level specification, mean correction, search for regular polynomials up to order 3 and for seasonal polynomials up to order 2 in the ARIMA order search, search for regular differences up to order 2 and for seasonal differences up to order 1 in the ARIMA order search, no adjustment for Easter effect, trading day adjustment taken into account by subtracting number of Saturdays and Sundays (weekends) from number of working days and multiplying the difference by $\frac{5}{2}$ and automatic detection and correction of four types of outliers (additive outliers, innovation outliers, temporary changes and level shifts).

list of 74 time series. We decided to use the rule of thumb (as was done by Arnoštová et al., 2011) and to exclude all indicators with a correlation coefficient of less than 0.35 in absolute terms from further analysis. In the end, we retrieve the final set of 35 time series, which are included in the medium-scale version of the DFM i.e. MDFM:

Table 2: Number of economic indicators by category

| Activity (survey) | Activity (hard data) | Employment | Financial conditions | Prices | Total |
|----------------------|-------------------------|------------|-------------------------|--------|-----------|
| 7 | 15 | 5 | 5 | 3 | 35 |

Note: Table shows the number of monthly/quarterly indicators applied in MDFM forecasting exercise.
Source: SORS, Eurostat, Bank of Slovenia.

The database reflects several important categories of macroeconomic variables, which are of particular relevance for factor extraction. The activity indicators are split into survey and hard data, where the former are mostly related to the relevant soft indicators (for instance various sentiment and confidence indicators) and the latter include the most important real variables (for instance real GDP growth, several IP indices, indices of turnover in services and retail trade). Employment aggregates contain data on employment and unemployment activity, money and financial market indicators employ Slovenian stock market index (SBITOP) and household loans activity, while in the group of prices one HICP and two different PPI indices are incorporated. Lastly, the considered indicators include also exogenous data (Brent Crude oil price, DAX stock market index, IP indices for Germany and Italy as two largest trading partners, and most important survey indicators for EA19 and Germany, such as for example consumer and industrial confidence indicators).

However, there exists one minor dilemma, as we should be aware that some properties of the estimators we consider in our FM are based on the assumption of a large set of indicators. In relation to that, the forecasting exercise of Boivin and Ng (2006) revealed that factors extracted from a smaller data set (around 40 variables) seem to do no worse or in many cases yield even better results than the ones extracted from larger data sets (more than 100 variables). Their results therefore suggest that sample size alone does not determine the properties of the estimates. The quality of the data must also be taken into account²⁰.

From the collected sample we are able to extract common factors that drive the dynamics of the specified macroeconomic database, using PCA. PCA is widely known as a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated series into a set of linearly

²⁰The efficiency of small-scale vs. large-scale FM is discussed in the next Subsection.

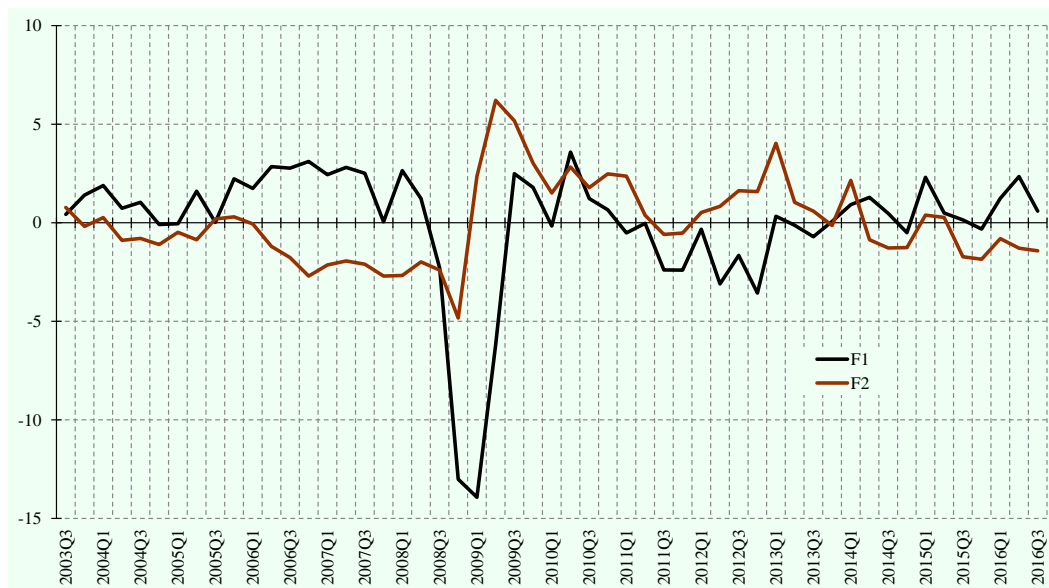
uncorrelated variables called principal components (PC). The above mentioned transformation is defined in such a way that the first PC has the largest possible variance (it accounts for as much of the variability in the data set as possible), and each succeeding component has a variance that is lower than the one of the preceding component (this is due to the criterion of orthogonal components). In the next step, factors are calculated using the extracted PCs. There are usually many ways of how to select an appropriate number of factors in the model. The most common include one of the following (Stock and Watson, 2011, 2016):

- The researcher arbitrarily sets a level of input variables' variance (roughly between 60 and 70%), and selects the smallest number of factors explaining at least that percentage of the variance;
- A visual diagnostic using scree plot, where the researcher either includes only those factors, whose corresponding eigenvalues of the covariance matrix are greater than 1 or finds the inflection point that separates high eigenvalues from low ones;
- Information criteria suggested by Bai and Ng (2002), where the criteria also allow for the specification of the lag order in the DFM;
- A prior assumption of a specified number of factors.

The first criterion suggests that 3-6 factors explain between 57.3% and 71.0 % of variation present in the data set. The second criterion shows that 9 factors have eigenvalues greater than 1, while a break point that separates high eigenvalues from low ones occurs between the second and the third factor which indicates a 2 FM solution. The third criterion displays Bai and Ng (2002) information criteria results, where the maximum number of factors considered is equal to the lower end from the first criteria i.e. 3 factors. The most relevant BIC3 criterion favours 2 factors, while all other (PC1, PC2, PC3, IPC1, IPC2 and IPC3) indicate a 3 FM. Moreover, we also estimate Equation 8 with 2 factors in order to analyse whether β_F have reasonable values. The results for the first window show p values for the first factor that are at the margin of significance, while in the case of the last window, beta coefficients for both factors are statistically significant. Lastly, we also check the forecasting ability of MDFMs with 1, 2, and 5 factors and find out that MDFM with 2 factors produce the most accurate one and two-quarter ahead forecasts²¹. Based on all of the aforementioned criteria we decided to follow a mix of suggestions and end up extracting 2 common factors. The following picture presents two factors at quarterly frequency, obtained by simply averaging monthly series, that are extracted via PCA:

²¹Results of factor analysis and forecast comparisons among different specifications of MDFMs are available in Appendix B.

Figure 1: Factors extracted from PCA



Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

As we already mentioned earlier, due to special statistical characteristics of the PCA, extracted factors lack a clear economic interpretation. Their atheoretical construction is also the reason why they are simply treated as patterns of change (unobservable components), which are common to all variables and thus reflect the variability of the set of predictors. Given the inability to clearly examine the nature of the factors, we are only able to conclude that the first factor reflects the high negative scale during the crisis episode, indicating strong co-movements of variables in economic downturn²². At a later stage, when we also simultaneously introduce second factor into the forecasting exercise, the elevated negative scale of the first factor is partially offset by the information content incorporated in the second factor. Finally, PCA is sensitive to the relative scaling of the original variables. In other words, using different transformation methods from those described on page 15 would lead to different factor estimates (different scales).

Finally we also briefly specify the structure of alternative models' data sets. In order to be able to discuss the efficiency of small-scale vs. large-scale FM and to compare their forecasting performance with our MDFM, the alternative set of models also includes the small-scale and large-scale versions of DFM (SDFM and LDFM). Furthermore, it should be pointed out that the same pre-adjustment steps (seasonal adjustment and different transformation methods) were considered before the estimation of each model, while the macroeconomic variables used were picked according to the literature:

²²Higher scale of first factor was for instance also obtained by Rusnák (2016).

- $AR(p) \Rightarrow$ baseline $AR(p)$ model is only based on real GDP growth, which implies that the $AR(p)$ forecast does not take into account any other monthly information flow;
- SDFM \Rightarrow SDFM utilizes the smaller subset of 10 time series that are the most correlated with the real GDP growth rate and applies only the first extracted factor to the state-space estimation procedure;
- LDFM \Rightarrow LDFM is based on the full data set that consists of 74 time series and applies the same number of extracted factors as the MDFM to the state-space estimation procedure;
- PC \Rightarrow PC model uses the same set of predictors as MDFM and therefore applies the same number of extracted factors to the OLS linear regression procedure;
- Bivariate OLS VAR(p) \Rightarrow only one variable from each aforementioned group of indicators (except prices) that exhibits the highest correlation with the real GDP growth is considered in the model. The Bivariate OLS VAR(p) thus contains six monthly indicators: real GDP growth (quadratically interpolated), IP index (section C (manufacturing) according to International Standard Classification of All Economic Activities (ISIC)), Slovenian sentiment indicator (survey), number of registered unemployed persons (taking into account all activities according to ISIC), SBITOP stock market index and the ECB main refinancing rate;
- BVAR(p) \Rightarrow here we consider only one BVAR(p) specification i.e. a medium-scale BVAR(p) model that utilizes the same set of indicators as Bivariate OLS VAR(p).

4.2 Results of pseudo out-of-sample forecasting exercise

In the last Subsection the forecasting performance of different competing models is evaluated. The evaluation is based on several standard goodness-of-fit (GoF) measures, which are commonly used to compare the forecasting performance of applied models:

- Mean absolute error (MAE) = $\frac{1}{T} \sum_{t=1}^T |y_t - y_{t,h}^f|$;
- Relative MAE (RMAE) = $\frac{\text{MAE}}{\text{MAE (AR}(p))}$;
- Root mean squared error (RMSE) = $\sqrt{\frac{1}{T} \sum_{t=1}^T (y_t - y_{t,h}^f)^2}$;

- Standardized RMSE (SRMSE) = $\frac{\text{RMSE}}{\sqrt{\frac{1}{T-1} \sum_{t=1}^T (y_t - \bar{y})^2}}$;
- Relative RMSE (RRMSE) = $\frac{\text{RMSE}}{\text{RMSE}(\text{AR}(p))}$;
- Theil inequality coefficient (TIC) = $\frac{\text{RMSE}}{\sqrt{\frac{1}{T} \sum_{t=1}^T y_t^2} + \sqrt{\frac{1}{T} \sum_{t=1}^T y_{t,h}^f}}^2$;
- Relative TIC (RTIC) = $\frac{\text{TIC}}{\text{TIC}(\text{AR}(p))}$;
- Modified Theil inequality coefficient (MTIC) = $\frac{\sqrt{\sum_{t=1}^T (y_{t,h}^f - y_t)^2}}{\sqrt{\sum_{t=1}^T y_t^2}}$;
- Relative MTIC (RMTIC) = $\frac{\text{MTIC}}{\text{MTIC}(\text{AR}(p))}$.

In the above equations, y_t presents the realized value of the real GDP growth in time t , $y_{t,h}^f$ denotes forecast of the variable at horizon h , based on the information set in time t , and T stands for the number of observations in the given period (Theil, 1958; Theil, 1966; Bliemel, 1973; Cimperman and Savšek, 2014).

In order to be able to simulate a forecasting exercise, the pseudo out-of-sample technique is applied, which divides the sample into an estimation sample and a forecasting sample. The former is also known as training sample and is used to estimate model parameters in order to be able to forecast "out-of-sample", while the latter is used to forecast real GDP growth and compare it to its realized value. As is already mentioned earlier, through the entire projection round an assumption is made that all economic indicators are readily available which leaves us with the realigned and balanced set of monthly indicators (similar to approach in Altissimo et al., 2010)²³.

When determining the pseudo out-of-sample forecasting method we follow the literature and adopt the rolling window strategy²⁴. It builds on the idea

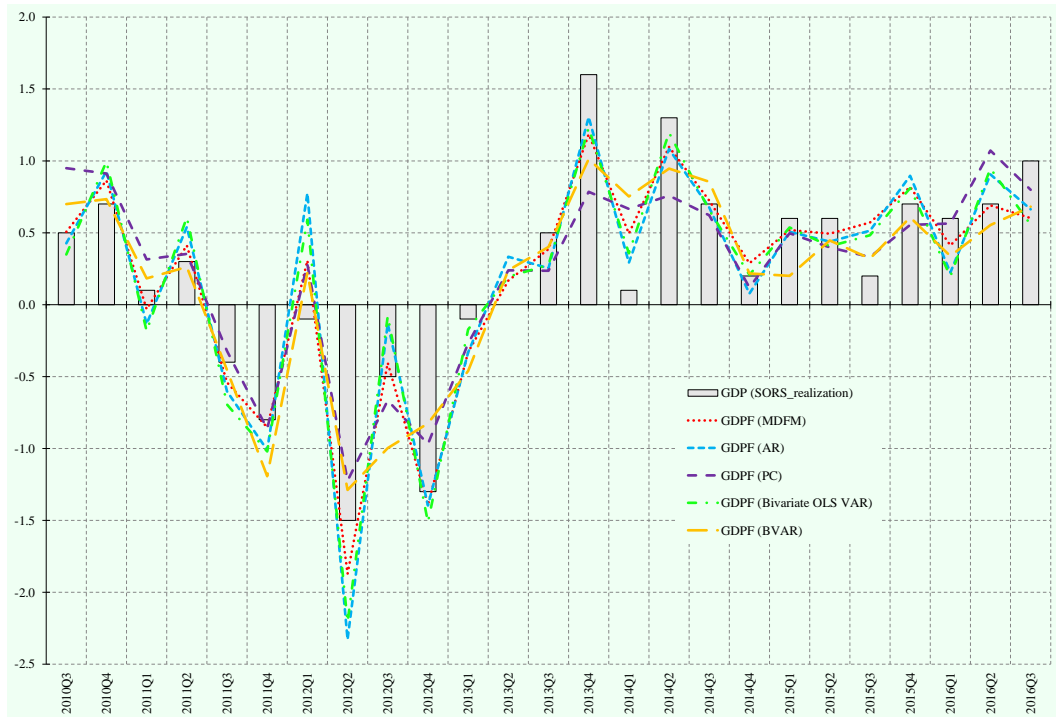
²³It should be noted that applied data have various publication lags. The full data set therefore becomes available 2 full months (around 8 weeks) after the beginning of the first quarter of the forecasting exercise.

²⁴An alternative method is called the recursive (expanding) window and is based on a increasing data window which always starts with the same initial observation.

that an initial sample from $t = 1, \dots, T$ is first used to determine a window width to estimate the model, while in the next stage, an h -step ahead out-of-sample forecast starting at time T is formed. After the first round of loops we move forward to $t + 1$, re-estimate the model using data from $t = 2, \dots, T + 1$ and perform the h -step ahead out-of-sample forecast starting at time $T + 1$. The same procedure repeats until the end of the available monthly data.

An advantage of the applied technique, when compared to the recursive (expanding) window approach, is that it is able to capture model specification uncertainty, model instability, and estimation uncertainty, in addition to the usual uncertainty of future events (Stock and Watson, 2008; Zivot, 2009)²⁵. In this paper, the estimation is based on a window width of 80 periods (months), meaning that forecasting sample starts in 2010 Q2 for one-quarter ahead forecasts and in 2010 Q3 for two-quarter ahead forecasts. In Figure 2, one-quarter ahead out-of-sample forecasts by all five applied models are presented:

Figure 2: One-step ahead real GDP growth forecasts



Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

²⁵Rolling window technique is, in comparison to the recursive (expanding) window approach, able to appropriately capture potential structural changes in parameter estimates caused by the global financial crisis. Given that our estimation window also includes such period, we can conclude that rolling window scheme is more convenient for application at hand.

The data plotted for the real GDP growth realization presents the latest accessible vintage. Since all obtained series (forecasts) are first constructed at a monthly frequency and then transformed to quarterly frequency, it can be seen that they reflect a close connection to short-term real GDP growth fluctuations. On the other side, potential higher frequency variations may also be relevant to detect medium and long term movements which can be especially helpful at identifying business cycle fluctuations. Furthermore, as is evident from the Figure 2, our one-step ahead MDFM forecast is effective in tracking real GDP growth, meaning that the model is well supported by the data. At the same time, MDFM performs most accurately at the horizons where other standard forecasting models have been shown to outperform a simple AR(p) model. This finding clearly reflects the fact that the use of monthly information matters most for the short-term horizon. In order to demonstrate it more clearly, the out-of-sample forecasting performance of the applied models is evaluated using different GoF measures.

Table 3 presents the pseudo out-of-sample GoF measures of one as well as of two-quarter ahead forecasts. A lower value of a particular GoF measure indicates that the underlying model produces more precise out-of-sample predictions in comparison to competing models. From the obtained results we are able to further analyse the forecasting ability of the five models and also to explicitly compare the forecasting performance of more sophisticated models to the one of the simple AR(p) model by using relative GoF measures. A value of relative GoF measures below 1 therefore indicates that the forecasts produced by the respective model are more precise than those of the benchmark AR(p) model:

Table 3: Goodness-of-fit measures for one and two-step ahead forecasts

| Measure/model | MDFM | | AR | | PC | | Biv. OLS VAR | | BVAR | |
|---------------|--------------|--------------|-------|-------|-------|-------|--------------|-------|-------|-------|
| | +1Q | +2Q | +1Q | +2Q | +1Q | +2Q | +1Q | +2Q | +1Q | +2Q |
| MAE | 0.179 | 0.190 | 0.268 | 0.271 | 0.247 | 0.220 | 0.264 | 0.273 | 0.251 | 0.239 |
| RMAE | 0.668 | 0.700 | 1.000 | 1.000 | 0.920 | 0.813 | 0.985 | 1.007 | 0.934 | 0.882 |
| RMSE | 0.222 | 0.229 | 0.331 | 0.336 | 0.308 | 0.285 | 0.315 | 0.326 | 0.306 | 0.291 |
| SRMSE | 0.306 | 0.314 | 0.455 | 0.463 | 0.424 | 0.392 | 0.434 | 0.448 | 0.421 | 0.400 |
| RRMSE | 0.672 | 0.679 | 1.000 | 1.000 | 0.933 | 0.847 | 0.954 | 0.969 | 0.926 | 0.864 |
| TIC | 0.148 | 0.153 | 0.209 | 0.212 | 0.219 | 0.207 | 0.201 | 0.207 | 0.215 | 0.205 |
| RTIC | 0.707 | 0.720 | 1.000 | 1.000 | 1.046 | 0.975 | 0.960 | 0.976 | 1.026 | 0.969 |
| MTIC | 0.297 | 0.306 | 0.442 | 0.450 | 0.412 | 0.381 | 0.422 | 0.436 | 0.409 | 0.389 |
| RMTIC | 0.672 | 0.679 | 1.000 | 1.000 | 0.933 | 0.847 | 0.954 | 0.969 | 0.926 | 0.864 |

Note: Numbers in bold indicate the lowest value for particular GoF measure among all models.

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

It can be seen, that the forecast errors beyond the first quarter generally tend to increase²⁶, but in comparison to other studies (for instance Arnoštová et al., 2011) the increases reported in Table 3 are not that substantial and are more in line with Bessonovs (2015). In general, the best forecasting performance is shown by the MDFM. Second best are the medium-scale BVAR(p) and PC models, which are quite closely followed by Bivariate OLS VAR(p) model. The forecasting abilities of all aforementioned models are relatively close to the top ranked model. AR(p) model is found to perform worst.

On the basis of the previous results we arrive to several interesting conclusions. First, all models that use additional monthly information - taking into account monthly information flow - exhibit a better forecasting performance compared to the benchmark AR(p) model. This is in line with the assumption that the information content in monthly indicators helps predicting the real GDP growth. Second, it can be seen that using solely the information from larger data sets (static factors) as regressors in the OLS framework results in similar performance to standard forecasting models but is not enough to dominate the BVAR(p) in forecasting accuracy (Rünstler et al., 2009 obtained similar results for several EA countries, while Liu et al., 2011 confirm such findings for Latin America EMEs). The MDFM however, is able to overcome this problem, as it incorporates two crucial characteristics that are needed to enhance forecasting accuracy over the alternative models: the first is reflected in the dynamic treatment of factors in the form of the VAR(p) model and the second is displayed in the application of the Kalman smoother and filter procedures. In comparison to previously mentioned models, smoothing and filtering procedures allow us to re-estimate the factors from PCA, and to update the short-term forecast of GDP once we have more information available in our model. This important finding shows that it is feasible to utilize the information content of a large number of series efficiently within the factor analysis framework. Third, the availability of monthly indicators for the current period appears to be promising at obtaining more accurate short-term forecasts of real GDP growth. The kind of the property is directly reflected in more involved models, since the existence of additional monthly observations allows the re-estimation of models and the updating of pseudo out-of-sample forecasts.

Lastly, it would be interesting to compare the forecasting performance of different DFM's specifications (SDFM, MDFM, and LDFM) in order to be able to directly relate to the discussion in Boivin and Ng (2006). Table 4 thus presents pseudo out-of-sample GoF measures for one and two-quarter ahead

²⁶The only exceptions are PC and BVAR(p) models, whose forecasting performances regarding the one and two-quarter ahead forecasts are actually improved. After performing the robustness check, where we computed the GoF measures for three-quarter ahead forecasts, we obtained values for both models that are in line with the general findings i.e. increase of forecast errors.

forecasts for different DFM specifications:

Table 4: Goodness-of-fit measures for one and two-step ahead forecasts for different DFM specifications

| Measure/model | SDFM | | MDFM | | LDFM | |
|---------------|-------|-------|--------------|--------------|-------|-------|
| | +1Q | +2Q | +1Q | +2Q | +1Q | +2Q |
| MAE | 0.197 | 0.205 | 0.179 | 0.190 | 0.183 | 0.193 |
| RMSE | 0.250 | 0.253 | 0.222 | 0.229 | 0.226 | 0.233 |
| SRMSE | 0.344 | 0.349 | 0.306 | 0.314 | 0.311 | 0.321 |
| TIC | 0.176 | 0.178 | 0.148 | 0.153 | 0.151 | 0.157 |
| MTIC | 0.334 | 0.339 | 0.297 | 0.306 | 0.302 | 0.312 |

Note: Numbers in bold indicate the lowest value for particular GoF measure among all DFM specifications.

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

The results show that factors extracted from a smaller data set (in our case 34 variables) yield better short-term forecasting results than LDFM (using 74 variables). The usual reasoning for this would be that, by adding a new series, the positive effect of additional information is overwhelmed by the effect of "oversampling" described by Boivin and Ng (2006)²⁷. Results further suggest that sample size alone does not determine the properties of the estimator. What should also be taken into account is the quality of the data. In comparison with the SDFM, we find out that too small samples bear too little information (in the case of SDFM we have only 10 variables) and are not able to provide better short-term forecasting performance compared to their larger counterparts. However, it should be stressed that all three specifications of FM exhibit better performance at short-term forecasting than the standard competing models considered in Table 3, again indicating that the additional information content captured in monthly series can be efficiently utilized within the framework of factor analysis.

We have also implemented the Diebold-Mariano test (D-M test) (Diebold and Mariano, 1995), using the Harvey et al. (1997) estimator of the variance of the difference between squared forecast errors, to check whether there exist statistically significant differences in the predictive abilities of the applied models²⁸. In a few cases the test results showed statistically insignificant differences between squared forecast errors. According to Ashley (2003), such results should be expected, as author pointed out that usually more than 100

²⁷This usually happens when the additional data are over correlated with the data from categories which are already included in factor estimation.

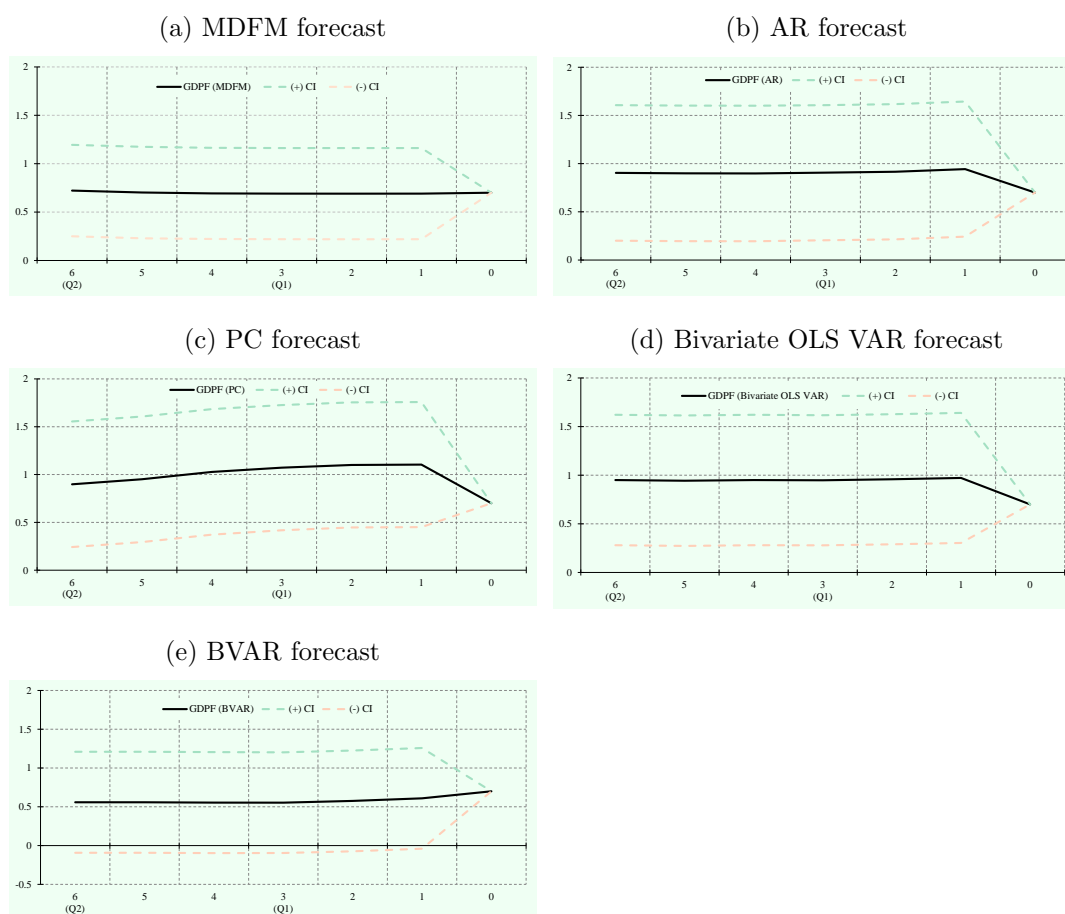
²⁸More formally, the D-M statistics tests the zero hypothesis of equal squared forecast errors and has a t -distribution with $N - 1$ degrees of freedom, where N stands for the number of forecast errors observed.

observations are needed to obtain statistically significant results whereas our evaluation sample includes only 25 observations²⁹.

The final exercise shows an example of the forecast for 2016 Q2 using all the implemented models. Since the realigned and balanced set of monthly indicators is used in our pseudo out-of-sample forecasting exercise, the results are only able to reflect how forecasts of larger models that also use more information evolve in comparison to a simple $AR(p)$ model. The first estimate corresponds to six months prior to the release of the real GDP growth data (the two-quarter ahead forecast), while the second one is evaluated just before the arrival of the official figures (the one-quarter ahead forecast). The last value in point 0 corresponds to the realized real GDP growth rate. In a given quarter, the most accurate predictions are obtained from MDFM and medium-scale $BVAR(p)$, which are relatively closely followed by Bivariate OLS $VAR(p)$ and $AR(p)$. The most imprecise forecasts are produced by the PC model. The main message and contribution of the exercise is to present the improvement of quarter-on-quarter real GDP growth forecasts of models that incorporate additional monthly information in comparison to simple $AR(p)$ model. To state it differently, the forecasting performance of more sophisticated models is improved within the observed quarter by acquiring more information from a larger set of monthly series.

²⁹This conclusion is also in line with one of the findings obtained by Harvey et al. (1997) in their Monte Carlo experiment, where authors stressed out that their modification of variance tends to produce better test results than standard D-M test, which are nevertheless to some extent still over-sized. Their finite sample modification and size correction procedure is thus an important, but not yet complete solution.

Figure 3: Model forecasts for 2016 Q2



Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

5 Conclusion

The current paper evaluates the out-of-sample forecasting performance for Slovenian quarterly real GDP growth using five types of models. The modelling framework applied ranges from a simple $AR(p)$ model, that is solely based on the real GDP growth, to more sophisticated structures that also incorporate, in a smaller (Bivariate OLS $VAR(p)$ and $BVAR(p)$ model) or larger (DFM and PC) extent, some additional monthly information. The selection of forecasting models follows the conclusions obtained by previous studies who generally found the usefulness of some of these models for short-term forecasting. The main goal of the paper is therefore to compare the forecasting performance of applied models and to answer the question whether the availability of additional monthly indicators improves the precision of out-of-sample forecasts of real GDP growth.

Based on the evolution of the exercise, the following basic conclusions are drawn. First, models that use monthly data outperform the standard $AR(p)$ model that uses only the information content in real GDP growth, and the forecasts obtained from other models tend to be increasingly more accurate as more information arrives within a quarter. Second, at all the horizons considered, the MDFM produces more accurate one and two-step ahead forecasts relative to other applied models. Next in ranking are generally found the $BVAR(p)$, PC, Bivariate OLS $VAR(p)$ and $AR(p)$. The top ranking of the MDFM is in line with previous advanced economy studies and indicates that FMs seem to be an efficient modelling procedure at capturing, integrating, and exploiting the information content of a large number of available indicators. To some extent this can be attributed to the signaling extraction process of DFM that is captured by the Kalman smoother and filter procedures. In comparison to other models considered in the paper, smoothing and filtering allow the re-estimation of the factors from PCA, resulting in an improvement in the accuracy of the short-term forecast of real GDP growth when more monthly information is available. However, robust D-M test results are, due to short sample limitations, not feasible, meaning that we can not resolve whether differences between squared forecast errors are statistically significant or not. Third, the availability of monthly indicators for the current period turns out most promising at obtaining more accurate short-term forecasts of real GDP growth, since the larger data set allows the re-estimation of models and the updating of pseudo out-of-sample forecasts. Lastly we can conclude that, in the case of Slovenia, smaller data sets (MDFM) are found to perform better and yield better short-term forecasting results than larger alternatives (LDFM) which goes along with the finding that sample size alone does not determine the properties of the estimator. What also plays an important role is the quality of data.

Regarding the application of MDFM forecasts in GDP forecasting procedure at the Bank of Slovenia, we can argue that the MDFM results could provide a good framework to support the bank's expert judgements and opinions when predicting the state of the economy on the short-term. For longer horizons, the MDFM results could serve as valid tool and a starting point in a structural macroeconometric model estimation procedure.

Finally, further research could focus on some important issues that were left open in the current paper. This especially holds for the assumption of readily available data set in our out-of-sample forecasting exercise, which needs to be relaxed in order to take into account the so called "ragged edge" data set problem. In that way the proper nowcasting exercise of Slovenian GDP could be conveyed. In addition to previously mentioned ideas, the forecasting performance of MDFM shall be compared to other state-of-the-art mixed frequency short-term forecasting models like MIDAS, mixed frequency VAR (MF-VAR), mixed frequency BVAR (MF-BVAR) and Three pass regression

filter (3PRF) models. It would be also interesting to compare forecasting accuracy of aforementioned models to some traditional nowcasting benchmark i. e. BE models.

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Appendices

A Full data set

Full set of economic indicators by category

| Series number | Series name | Series name in model | Type* | Correlation with GDP | Transformation method** | Included in SDFM!*** | Included in MDFM!**** |
|---------------|--|-----------------------|--------|----------------------|-------------------------|----------------------|-----------------------|
| 1 | Real GDP growth | BDFG | H | 1.000 | 0 | Y | Y |
| 2 | Brent crude oil price (in EUR) | BRENT_EUR | F (EX) | 0.365 | 2 | N | Y |
| 3 | HICP exc. alcoholic beverage and tobacco | HICP_B.ALKO.TOBAK | P | 0.338 | 3 | N | N |
| 4 | HICP (goods) | HICP_BLAGO | P | 0.301 | 3 | N | N |
| 5 | Core inflation | HICP_CORE | P | 0.152 | 3 | N | N |
| 6 | HICP (total) | HICP_SKUP | P | 0.273 | 3 | N | Y |
| 7 | DAX stock market index | DAX | F (EX) | 0.392 | 2 | N | Y |
| 8 | Persons in employment (total) | DEL_AKTIVNI | E | 0.560 | 2 | N | Y |
| 9 | ECB main refinancing rate | ECB_OBR_MER_REF | F (EX) | 0.650 | 1 | Y | Y |
| 10 | EUR/GDP exch. rate | EUR_GDP | F (EX) | -0.213 | 2 | N | N |
| 11 | EUR/HRK exch. rate | EUR_HRK | F (EX) | -0.284 | 2 | N | N |
| 12 | EUR/USD exch. rate | EUR_USD | F (EX) | 0.278 | 2 | N | N |
| 13 | Value of new contracts in construction | GRADEL_NOVA | H | -0.045 | 2 | N | N |
| 14 | Value of construction put in place in construction | GRADEL_SKUP | H | 0.326 | 2 | N | N |
| 15 | ILO unemployment rate | ILO_BREZPO | E | -0.349 | 1 | N | Y |
| 16 | Industrial production index (IPI) (total industry) | IP_ICD | H | 0.647 | 2 | Y | Y |
| 17 | IPI (mining and quarrying) | IP_RUDAR | H | -0.039 | 2 | N | N |
| 18 | IPI (manufacturing) | IP_PREDEL | H | 0.661 | 2 | Y | Y |
| 19 | IPI (electricity, gas and water supply) | IP_OSK_EL_PLIN | H | 0.157 | 2 | N | N |
| 20 | IPI (intermediate goods industries) | IP_SUROV | H | 0.654 | 2 | Y | Y |
| 21 | IPI (energy related industries) | IP_ENERG | H | 0.078 | 2 | N | N |
| 22 | IPI (intermediate goods industries excl. energy) | IP_PROIZ_VMES_PORAB | H | 0.603 | 2 | Y | Y |
| 23 | IPI (capital goods industries) | IP_PROIZ_INVEST | H | 0.411 | 2 | N | Y |
| 24 | IPI (consumer goods industries) | IP_PROIZ_SIR_PORAB | H | 0.368 | 2 | N | Y |
| 25 | IPI (durable consumer goods industries) | IP_TRAIN_PROIZ | H | 0.401 | 2 | N | Y |
| 26 | IPI (non-durable consumer goods industries) | IP_NETRAIN_PROIZ | H | 0.294 | 2 | N | Y |
| 27 | IPI (German industry) | IP_DE | H (EX) | 0.544 | 2 | N | Y |
| 28 | IPI (Italian industry) | IP_IT | H (EX) | 0.640 | 2 | Y | Y |
| 29 | Sentiment indicator | KAZ_KLIMA | S | 0.509 | 1 | N | Y |
| 30 | Industrial confidence indicator (Slovenia) | KAZ_ZAUP_PODJET_SI | S | 0.282 | 1 | N | N |
| 31 | Confidence indicator in manufacturing | KAZ_ZAUP_PREDEL | S | 0.290 | 1 | N | N |
| 32 | Confidence indicator in retail trade | KAZ_ZAUP_TRGO_DROBNO | S | 0.325 | 1 | N | N |
| 33 | Consumer confidence indicator | KAZ_ZAUP_POTRO | S | 0.335 | 1 | N | N |
| 34 | Confidence indicator in services | KAZ_ZAUP_STORIT | S | 0.506 | 1 | N | Y |
| 35 | Confidence indicator in construction | KAZ_ZAUP_GRADB | S | 0.416 | 1 | N | Y |
| 36 | Business tendency in manufacturing (production) | KAZ_PREDEL_PROIZ | S | 0.138 | 1 | N | N |
| 37 | Business tendency in manufacturing (production expectations) | KAZ_PREDEL_PROIZ | S | 0.109 | 1 | N | N |
| 38 | Business tendency in retail trade (business situation) | KAZ_TRGO_DROBNO_POLOZ | S | 0.287 | 1 | N | N |
| 39 | Business tendency in retail trade (sales) | KAZ_TRGO_DROBNO_PROD | S | 0.285 | 1 | N | N |
| 40 | Business tendency in retail trade (expected sales) | KAZ_TRGO_DROBNO_PROD | S | 0.118 | 1 | N | N |
| 41 | Consumer survey (general economic situation in Slovenia over the past 12 months) | KAZ_POTRO_STANJE | S | 0.544 | 1 | N | N |
| 42 | Consumer survey (general economic situation in Slovenia over the next 12 months) | KAZ_POTRO_STANJE | S | 0.256 | 1 | N | N |
| 43 | Business tendency in services (business situation) | KAZ_STORIT_POLOZ | S | 0.379 | 1 | N | Y |
| 44 | Business tendency in construction (assessment of building activity) | KAZ_GRADB_GRADEL | S | 0.253 | 1 | N | N |
| 45 | Industrial confidence indicator (EA19) | KAZ_ZAUP_PODJET_EA19 | S (EX) | 0.401 | 1 | N | Y |
| 46 | Industrial confidence indicator (Germany) | KAZ_ZAUP_PODJET_DE | S (EX) | 0.438 | 1 | N | Y |
| 47 | Consumer confidence indicator (EA19) | KAZ_ZAUP_POTRO_EA19 | S (EX) | 0.333 | 1 | N | N |
| 48 | Consumer confidence indicator (Germany) | KAZ_ZAUP_POTRO_DE | S (EX) | 0.449 | 1 | N | Y |
| 49 | Short-term deposits | KRATK_DEPOZIT | F | -0.030 | 2 | N | N |
| 50 | M1 | M1 | F | 0.197 | 2 | N | N |
| 51 | M3 | M3 | F | 0.170 | 2 | N | N |
| 52 | Balance of payments (goods account) | PB_SALDO_BLAGO | H | -0.207 | 1 | N | N |
| 53 | Balance of payments (services account) | PB_SALDO_STORIT | H | 0.115 | 1 | N | N |
| 54 | Loans to households | POSOL_GOSPOD | F | 0.383 | 2 | N | Y |
| 55 | Loans to non-financial companies | POSOL_NEFIN_DRUZ | F | 0.212 | 2 | N | N |
| 56 | Loans to private sector | POSOL_ZASEB_SEK | F | 0.278 | 2 | N | N |
| 57 | Producer price index (PPI) (total exc. construction, sewerage, waste management and remediation activities) | PPI_SKUP | P | 0.247 | 3 | N | Y |
| 58 | PPI (mining and quarrying) | PPI_RUDAR | P | 0.006 | 3 | N | N |
| 59 | PPI (manufacturing) | PPI_PREDEL | P | 0.363 | 3 | N | Y |
| 60 | PPI (intermediate goods industries) | PPI_SUROV | P | 0.337 | 3 | N | N |
| 61 | PPI (energy related industries) | PPI_ENERG | P | -0.020 | 3 | N | N |
| 62 | PPI (capital goods industries) | PPI_PROIZ_INVEST | P | 0.077 | 3 | N | N |
| 63 | PPI (consumer goods industries) | PPI_PROIZ_SIR_PORAB | P | 0.333 | 3 | N | N |
| 64 | PPI (durable consumer goods industries) | PPI_TRAIN_BLAGO | P | 0.124 | 3 | N | N |
| 65 | PPI (non-durable consumer goods industries) | PPI_NETRAIN_BLAGO | P | 0.318 | 3 | N | N |
| 66 | Self employed persons (total) | SAMOZAPO_OSEBE | E | 0.118 | 2 | N | N |
| 67 | SBI TOP stock market index | SBI_TOP | F | 0.521 | 2 | N | Y |
| 68 | Number of registered unemployed (total) | ST_REG_BREZPO | E | -0.621 | 2 | Y | Y |
| 69 | Registered rate of unemployment (total) | STOP_REG_BREZPO | E | -0.575 | 1 | N | Y |
| 70 | Value index of turnover in services activities (total) | STORIT_SKUP | H | 0.443 | 2 | N | Y |
| 71 | Volume index of turnover in retail trade (retail trade) | TRGO_DROBNO | H | 0.493 | 2 | N | Y |
| 72 | Volume index of turnover in retail trade (wholesale and retail trade and repair of motor vehicles and motorcycles) | TRGO_MOTOR | H | 0.602 | 2 | Y | Y |
| 73 | Volume index of turnover in retail trade (total) | TRGO_SKUP | H | 0.619 | 2 | Y | Y |
| 74 | Persons in paid employment (total) | ZAPO_OSEBE | E | 0.533 | 2 | N | Y |

Note: * H - Hard data, S - Survey, E - Employment, F - Financial conditions, P - Prices. Label (EX) marks foreign indicators.

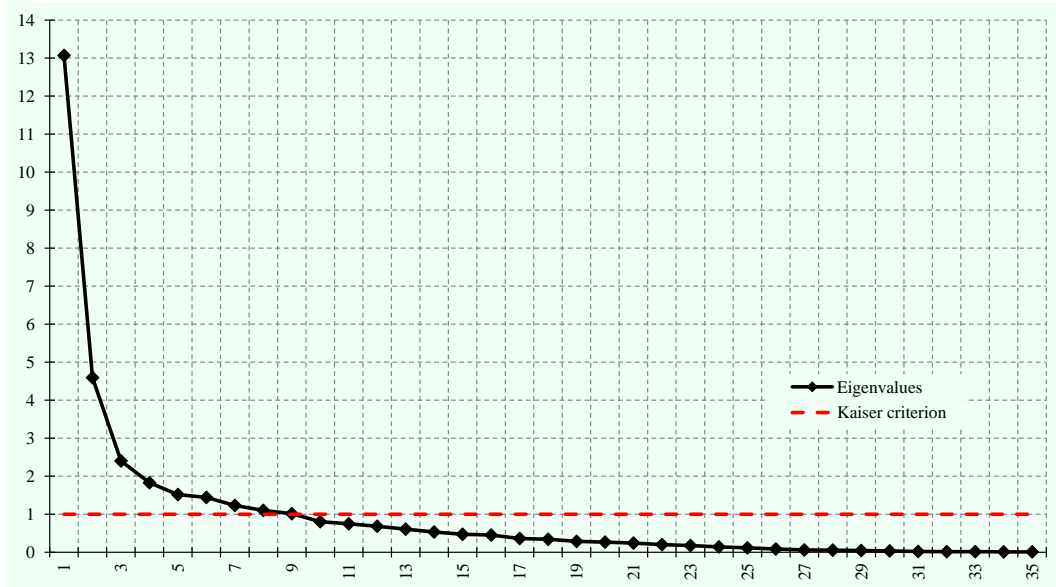
** 0 - No transformation, 1 - Three-month difference, 2 - Three-month growth rate, 3 - Three-month difference of yearly growth rate.

*** Y - Yes, N - No.

Source: SORS, Eurostat, Bank of Slovenia.

B Factor analysis

Scree plot



Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

Bai and Ng (2002) criteria results

| Number of factors/measure | PC1 | PC2 | PC3 | IPC1 | IPC2 | IPC3 | AIC3 | BIC3 |
|---------------------------|---------------|---------------|---------------|--------------|--------------|--------------|---------------|---------------|
| 3 | 17.357 | 17.624 | 16.762 | 2.904 | 2.925 | 2.858 | 15.493 | 24.255 |
| 2 | 18.846 | 18.933 | 18.449 | 2.997 | 3.010 | 2.966 | 17.612 | 23.485 |
| 1 | 22.680 | 22.831 | 22.481 | 3.175 | 3.177 | 3.155 | 22.068 | 25.019 |

Note: Numbers in bold indicate the lowest value for each measure (optimal number of factors).

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

PC model: first and last window estimates

| Dependent variable: BDPG | | |
|--------------------------|---------------------|---------------------|
| Regressor | Coefficient (FW) | Coefficient (LW) |
| C | 0.043 (0.030) | 0.052* (0.020) |
| BDPG(-1) | 0.814*** (0.112) | 0.510*** (0.081) |
| F1 | 0.019 (0.012) | 0.068*** (0.013) |
| F2 | 0.020* (0.012) | -0.020* (0.012) |
| Observations | 80 | 80 |
| R^2 | 0.883 | 0.725 |
| Adjusted R^2 | 0.878 | 0.714 |
| Log-likelihood | 20.936 | 36.806 |
| F-statistic | 188.341 | 66.692 |

Note: Standard errors in parentheses. FW and LW indicate first and last window estimates.
 *** Significant at the 1% level, ** Significant at the 5% level, * Significant at the 10% level.
Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

Comparing forecasting performance of MDFMs using various number of factors

| Measure/model | MDFM 1 | | MDFM 2 | | MDFM 5 | |
|---------------|--------|-------|--------------|--------------|--------|-------|
| | +1Q | +2Q | +1Q | +2Q | +1Q | +2Q |
| MAE | 0.193 | 0.202 | 0.179 | 0.190 | 0.195 | 0.202 |
| RMSE | 0.246 | 0.249 | 0.222 | 0.229 | 0.238 | 0.242 |
| SRMSE | 0.338 | 0.343 | 0.306 | 0.314 | 0.327 | 0.333 |
| TIC | 0.173 | 0.175 | 0.148 | 0.153 | 0.162 | 0.165 |
| MTIC | 0.329 | 0.333 | 0.297 | 0.306 | 0.318 | 0.324 |

Note: Numbers in bold indicate the lowest value for particular GoF measure among all MDFM specifications. MDFM 1, 2 and 5 stand for one, two and five FM.
Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

C Last window model estimates³⁰

MDFM

Smoothed factor VAR

| Lags/variable | F1(S) | F2(S) |
|----------------|----------------------|----------------------|
| F1(S)(-1) | 0.789*** (0.100) | -0.054*** (0.014) |
| F1(S)(-2) | 0.203* (0.149) | 0.051** (0.021) |
| F1(S)(-3) | -0.521*** (0.151) | -0.031* (0.021) |
| F1(S)(-4) | 0.453*** (0.113) | -0.002 (0.016) |
| F2(S)(-1) | 1.193* (0.744) | 1.150*** (0.103) |
| F2(S)(-2) | 0.107 (1.094) | -0.221* (0.151) |
| F2(S)(-3) | -0.228 (1.091) | -0.337** (0.151) |
| F2(S)(-4) | -0.068 (0.674) | 0.310*** (0.093) |
| Observations | Whole sample | |
| R^2 | 0.780 | 0.836 |
| Adjusted R^2 | 0.769 | 0.828 |
| Log-likelihood | -312.341 | -5.492 |
| F-statistic | 74.315 | 106.919 |

Note: Standard errors in parentheses. Estimates of Smoothed factor VAR coefficients are based on the entire sample and enter as starting values in the final Kalman filter procedure (F1(S) and F2(S) indicate smoothed factor estimates).

*** Significant at the 1% level, ** Significant at the 5% level, * Significant at the 10% level.

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

³⁰PC model estimates are available in Appendix B.

MDFM estimates

| Dependent variable: BDPG | |
|--------------------------|-------------------|
| Regressor | Coefficient |
| F1(S) | 0.063* (0.035) |
| F2(S) | -0.024 (1.442) |
| Observations | 80 |
| Log-likelihood | 4.365 |

Note: Standard errors in parentheses. Coefficients are obtained from the final step of Kalman filter procedure (F1(S) and F2(S) indicate smoothed factor estimates).

*** Significant at the 1% level, ** Significant at the 5% level, * Significant at the 10% level.

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

AR model

AR model estimates

| Lags/variable | BDPG |
|----------------|----------------------|
| C | 0.009 (0.011) |
| BDPG(-1) | 1.408*** (0.102) |
| BDPG(-2) | -0.292** (0.179) |
| BDPG(-3) | -1.399*** (0.178) |
| BDPG(-4) | 1.603*** (0.155) |
| BDPG(-5) | -0.223 (0.181) |
| BDPG(-6) | -0.645*** (0.178) |
| BDPG(-7) | 0.506*** (0.099) |
| Observations | 80 |
| R^2 | 0.912 |
| Adjusted R^2 | 0.904 |
| Log-likelihood | 82.559 |
| F-statistic | 106.989 |

Note: Standard errors in parentheses.

*** Significant at the 1% level, ** Significant at the 5% level, * Significant at the 10% level.

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

Bivariate OLS VAR model³¹

Bivariate OLS VAR model estimates: BDPG and IP_PREDEL

| Lags/variable | BDPG | IP_PREDEL |
|----------------|----------------------|----------------------|
| C | 0.007 (0.013) | 0.265 (0.204) |
| BDPG(-1) | 1.135*** (0.082) | 3.982*** (1.152) |
| BDPG(-2) | -0.136 (0.121) | -2.831* (1.699) |
| BDPG(-3) | -0.857*** (0.123) | -0.036 (1.731) |
| BDPG(-4) | 0.765*** (0.085) | 1.543 (1.199) |
| IP_PREDEL(-1) | 0.002 (0.008) | 0.625*** (0.111) |
| IP_PREDEL(-2) | 0.010 (0.008) | 0.049 (0.119) |
| IP_PREDEL(-3) | -0.006 (0.008) | -0.622*** (0.118) |
| IP_PREDEL(-4) | -0.001 (0.008) | 0.305*** (0.105) |
| Observations | 80 | 80 |
| R^2 | 0.867 | 0.620 |
| Adjusted R^2 | 0.852 | 0.577 |
| Log-likelihood | 65.934 | -145.656 |
| F-statistic | 57.904 | 14.460 |

Note: Standard errors in parentheses.

*** Significant at the 1% level, ** Significant at the 5% level, * Significant at the 10% level.

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

³¹A few estimated coefficients turn out insignificant. According to Keating (2000) such results are acceptable in a forecasting exercise but on the other hand cause the impulse responses and variance decompositions to often be imprecisely determined.

Bivariate OLS VAR model estimates: BDPG and KAZ_KLIMA

| Lags/variable | BDPG | KAZ_KLIMA |
|----------------|----------------------|----------------------|
| C | 0.004 (0.011) | 0.222 (0.360) |
| BDPG(-1) | 1.368*** (0.105) | 0.712 (3.468) |
| BDPG(-2) | -0.266* (0.182) | 6.388 (6.028) |
| BDPG(-3) | -1.468*** (0.185) | -9.475* (6.109) |
| BDPG(-4) | 1.626*** (0.159) | 2.509 (5.237) |
| BDPG(-5) | -0.196 (0.183) | 9.660* (6.036) |
| BDPG(-6) | -0.726*** (0.184) | -10.117* (6.087) |
| BDPG(-7) | 0.546*** (0.104) | 2.283 (3.448) |
| KAZ_KLIMA(-1) | 0.004 (0.004) | 0.920*** (0.123) |
| KAZ_KLIMA(-2) | 0.005 (0.005) | -0.283** (0.164) |
| KAZ_KLIMA(-3) | -0.004 (0.005) | -0.397*** (0.151) |
| KAZ_KLIMA(-4) | 0.002 (0.004) | 0.592*** (0.137) |
| KAZ_KLIMA(-5) | 0.003 (0.005) | -0.450*** (0.149) |
| KAZ_KLIMA(-6) | -0.001 (0.005) | -0.013 (0.155) |
| KAZ_KLIMA(-7) | 0.003 (0.003) | 0.151* (0.112) |
| Observations | 80 | 80 |
| R^2 | 0.923 | 0.646 |
| Adjusted R^2 | 0.907 | 0.570 |
| Log-likelihood | 87.855 | -191.978 |
| F-statistic | 55.788 | 8.477 |

Note: Standard errors in parentheses.

*** Significant at the 1% level, ** Significant at the 5% level, * Significant at the 10% level.

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

Bivariate OLS VAR model estimates: BDPG and ST_REG_BREZPO

| Lags/variable | BDPG | ST_REG_BREZPO |
|-------------------|----------------------|----------------------|
| C | 0.011 (0.013) | 0.012 (0.089) |
| BDPG(-1) | 1.431*** (0.108) | -0.890 (0.750) |
| BDPG(-2) | -0.473*** (0.143) | 0.761 (0.991) |
| BDPG(-3) | -0.819*** (0.110) | 0.118 (0.762) |
| BDPG(-4) | 1.133*** (0.141) | -0.713 (0.981) |
| BDPG(-5) | -0.361*** (0.109) | 0.221 (0.756) |
| ST_REG_BREZPO(-1) | -0.014 (0.017) | 1.608*** (0.117) |
| ST_REG_BREZPO(-2) | 0.037 (0.031) | -0.875*** (0.213) |
| ST_REG_BREZPO(-3) | -0.063** (0.034) | -0.180 (0.236) |
| ST_REG_BREZPO(-4) | 0.083*** (0.030) | 0.541*** (0.211) |
| ST_REG_BREZPO(-5) | -0.044*** (0.016) | -0.188** (0.109) |
| Observations | 80 | 80 |
| R^2 | 0.896 | 0.918 |
| Adjusted R^2 | 0.881 | 0.906 |
| Log-likelihood | 75.746 | -79.337 |
| F-statistic | 59.451 | 77.413 |

Note: Standard errors in parentheses.

*** Significant at the 1% level, ** Significant at the 5% level, * Significant at the 10% level.

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

Bivariate OLS VAR model estimates: BDPG and SBI.TOP

| Lags/variable | BDPG | SBI.TOP |
|----------------|----------------------|----------------------|
| C | 0.016* (0.011) | -0.256 (0.472) |
| BDPG(-1) | 1.399*** (0.106) | -0.274 (4.462) |
| BDPG(-2) | -0.233 (0.184) | 8.910 (7.731) |
| BDPG(-3) | -1.531*** (0.185) | -8.270 (7.786) |
| BDPG(-4) | 1.645*** (0.170) | -2.586 (7.143) |
| BDPG(-5) | -0.127 (0.188) | 9.522 (7.894) |
| BDPG(-6) | -0.750*** (0.185) | -9.724 (7.756) |
| BDPG(-7) | 0.540*** (0.102) | 2.321 (4.295) |
| SBI.TOP(-1) | 0.003 (0.003) | 1.166*** (0.115) |
| SBI.TOP(-2) | 0.002 (0.004) | -0.304** (0.170) |
| SBI.TOP(-3) | -0.005 (0.004) | -0.310** (0.171) |
| SBI.TOP(-4) | 0.002 (0.004) | 0.214 (0.173) |
| SBI.TOP(-5) | 0.003 (0.004) | 0.282* (0.171) |
| SBI.TOP(-6) | 0.000 (0.004) | -0.524*** (0.163) |
| SBI.TOP(-7) | -0.001 (0.003) | 0.294*** (0.105) |
| Observations | 80 | 80 |
| R^2 | 0.922 | 0.784 |
| Adjusted R^2 | 0.906 | 0.737 |
| Log-likelihood | 87.433 | -211.607 |
| F-statistic | 55.153 | 16.799 |

Note: Standard errors in parentheses.

*** Significant at the 1% level, ** Significant at the 5% level, * Significant at the 10% level.

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

Bivariate OLS VAR model estimates: BDPG and ECB_OBR_MER_REF

| Lags/variable | BDPG | ECB_OBR_MER_REF |
|---------------------|----------------------|----------------------|
| C | 0.004 (0.014) | -0.010* (0.007) |
| BDPG(-1) | 1.116*** (0.080) | 0.033 (0.040) |
| BDPG(-2) | -0.102 (0.121) | -0.047 (0.060) |
| BDPG(-3) | -0.822*** (0.121) | 0.086* (0.060) |
| BDPG(-4) | 0.751*** (0.081) | -0.041 (0.040) |
| ECB_OBR_MER_REF(-1) | -0.297 (0.235) | 1.423*** (0.116) |
| ECB_OBR_MER_REF(-2) | 0.117 (0.406) | -0.694*** (0.201) |
| ECB_OBR_MER_REF(-3) | 0.118 (0.403) | -0.145 (0.199) |
| ECB_OBR_MER_REF(-4) | -0.034 (0.229) | 0.213** (0.113) |
| Observations | 80 | 80 |
| R^2 | 0.869 | 0.838 |
| Adjusted R^2 | 0.854 | 0.820 |
| Log-likelihood | 66.564 | 123.079 |
| F-statistic | 58.963 | 45.898 |

Note: Standard errors in parentheses.

*** Significant at the 1% level, ** Significant at the 5% level, * Significant at the 10% level.

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

BVAR model³²

BVAR model estimates

| Lags/variable | BDPG | IP_PREDEL | KAZ_KLIMA | ST_REG_BREZPO | SBLTOP | ECB_OBR_MER_REF |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| C | 0.013 (0.023) | 0.215 (0.254) | 0.046 (0.452) | 0.042 (0.123) | -0.798* (0.557) | -0.006 (0.010) |
| BDPG(-1) | 0.791*** (0.076) | 1.018*** (0.742) | 0.219 (1.300) | -0.098 (0.350) | 1.651 (1.584) | 0.026 (0.028) |
| IP_PREDEL(-1) | -0.000 (0.007) | 0.564*** (0.094) | 0.122 (0.134) | -0.027 (0.037) | 0.079 (0.165) | -0.001 (0.003) |
| KAZ_KLIMA(-1) | 0.003 (0.004) | -0.014 (0.042) | 0.632*** (0.092) | -0.029* (0.020) | -0.011 (0.092) | 0.002 (0.002) |
| ST_REG_BREZPO(-1) | 0.001 (0.008) | -0.065 (0.091) | 0.176 (0.162) | 0.876*** (0.047) | -0.030 (0.199) | -0.002 (0.004) |
| SBLTOP(-1) | -0.001 (0.002) | -0.004 (0.027) | 0.059 (0.049) | 0.014 (0.013) | 0.795*** (0.067) | 0.002* (0.001) |
| ECB_OBR_MER_REF(-1) | -0.059 (0.140) | -0.690 (1.557) | -2.212 (2.793) | 0.751 (0.753) | -7.771** (3.420) | 0.835*** (0.068) |
| Observations | 80 | 80 | 80 | 80 | 80 | 80 |
| R^2 | 0.626 | 0.363 | 0.420 | 0.837 | 0.691 | 0.686 |
| Adjusted R^2 | 0.595 | 0.310 | 0.373 | 0.823 | 0.666 | 0.660 |
| F-statistic | 20.329 | 6.920 | 8.817 | 62.320 | 27.261 | 26.603 |

Note: Standard errors in parentheses.

**** Significant at the 1% level, ** Significant at the 5% level, * Significant at the 10% level.*

Source: Own calculations, SORS, Eurostat, Bank of Slovenia.

³²The same reasoning as in the case of Bivariate OLS VAR model also holds for BVAR model.